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LETTER TO THE EDITOR

New Virasoro and Kac–Moody symmetries for the non-linear σ-model

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Abstract. In this letter, we give new linearisation equations for the O(3) non-linear σ-model which are similar to those obtained in the stationary axially symmetric Einstein field equations. By using the linearisation equations, we confirm the existence of the infinite-dimensional symmetries, which are dependent on a spectrum parameter, in the O(3) non-linear σ-model. The relationships between our hidden transformations and the infinitesimal Riemann–Hilbert transformation are discussed.

For the principal chiral model, there exists a transformation of the hidden symmetry with a spectrum parameter in a Lax pair to leave the equation of motion invariant [1]. By expanding in powers of the spectrum parameter, the algebra of the transformation is known to be the Kac–Moody algebra without a central extension [2]. Further investigation shows that the hidden transformation originates from the Riemann–Hilbert transformation which was first introduced by Zakharov and Milkhailov [3], and the relationships between these transformations have been discussed in [4]. However, in addition to the transformation structure [5] arising from the Geroch symmetry [6] there is also found another type of hidden transformation [7] related to the Virasoro symmetry which has received much attention recently [8] for the stationary axially symmetric Einstein field equations. This motivates us to search for similar structures of other models.

Although the hidden transformation analogue to the above for the non-linear σ-model was given in [9] the connection between the hidden transformation and the Riemann–Hilbert transformation involving the Kac–Moody symmetry was not made clear [10], and no transformation was found to correspond to the Virasoro symmetry. The purpose of this letter is to construct such transformations that can be used to confirm the existence of the Kac–Moody symmetry and the Virasoro symmetry for the non-linear σ-model.

We should indicate that the situation considered here is entirely different from that of the quantum field discussed by Witten and others [8, 11]. In the latter case the operators of the Kac–Moody algebra are extracted from the quantised commutators of the current–current which contain the Schwinger term and the operators of the Virasoro algebra are constructed from moments of the stress–energy tensor. The central extensions of both the algebras emerge naturally from the Schwinger term and the trace anomaly. The necessary and sufficient condition for the existence of both the infinite-dimensional symmetries is that the system must possess the Wess–Zumino term.

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Nevertheless, our symmetries are independent of the Wess-Zumino term and the Poisson bracket relation. In our case, the operators T_n^a and L_n of the Kac-Moody algebra and the Virasoro algebra are defined as the classical transformations of Lie algebras, which operate in the tangent space of the solution manifold and generate new solutions from the known one for the equation of motion. It is easy to see that L_0 is unbounded above and below and the unitary conditions are not satisfied by the operators T_n^a and L_n so that we can establish the non-unitary and non-highest weight representations of both the Kac-Moody algebra and the Virasoro algebra without the central extensions. Readers should note these differences between the quantum level and the classical level.

This letter is organised as follows. We present the self-dual equations of a complex potential for the O(3) non-linear σ-model and then derive the linearisation equations similar to the Hauser-Ernst equations in order to replace the usual Lax pair [9]. We verify the integrability of the linearisation equations by the existence of the self-dual equations. Two infinitesimal transformations are then given and invariance of the self-dual equations under these transformations is proved. Both transformations are then reformulated as infinitesimal Riemann-Hilbert transformations. With the new formalism it is not difficult to calculate the commutators of the transformations that evidently form the semidirect product of the Kac-Moody algebra and the Virasoro algebra.

For the O(3) non-linear σ-model, the Lagrangian has the form

$$\mathscr{L}=\tfrac{1}{8}\operatorname{Tr}(\partial_\mu N(x)\partial^\mu N(x)) \qquad \mu=0,1 \tag{1}$$

with the constraint

$$N^2(x)=\alpha^2 I \tag{2}$$

where α is a real constant, I is a 2×2 unit matrix and

$$N(x)=\sum_{a=1}^{3} N^a(x)\sigma_a \qquad N^+(x)=N(x) \tag{3}$$

and $\sigma_a (a=1,2,3)$ are the Pauli matrices, $^+$ denotes the Hermitian conjugate and $\partial_\mu=\partial/\partial x^\mu$ $(\mu=0,1)$. According to the Lagrangian we induce the equation of motion

$$\partial_\mu(N\partial^\mu N)=0. \tag{4}$$

From the above equation, the twist potential X is defined as

$$\partial_\mu X=\alpha^{-1}\varepsilon_{\mu\nu}N\partial^\nu N \tag{5}$$

where $\varepsilon_{01}=-\varepsilon_{10}=1$ and $\varepsilon_{00}=\varepsilon_{11}=0$. Because of the feature of the Hermiticity for N, we have

$$\partial_\mu(X+X^+)=0. \tag{6}$$

If one chooses $\operatorname{Tr} X=2\beta$ where β is a real constant, one can obtain a relation

$$X^++X=2\beta I \tag{7}$$

where I is a 2×2 unit matrix. In general, $\alpha=1$ and $\beta=0$ [9] but we need not impose such restrictions on α and β for our purpose. We shall see it is helpful to express the Virasoro symmetry of the model in terms of α and β.

Now we derive the self-dual equations

$$2\beta\partial_\mu E+2\alpha\varepsilon_{\mu\nu}\partial^\nu E=(E+E^+)\partial_\mu E \tag{8}$$

by introducing a complex potential

$$E = N + X.$$

If the dual operation is defined as

$$^*\mathrm{d}\xi = \mathrm{d}\xi \qquad ^*\mathrm{d}\eta = -\mathrm{d}\eta \tag{9}$$

in the light-cone coordinates $\xi = \frac{1}{2}(x^0 + x^1)$ and $\eta = \frac{1}{2}(x^0 - x^1)$, equation (8) can be re-expressed in differential form

$$2(\beta + \alpha^*)\,\mathrm{d}E = (E + E^+)\,\mathrm{d}E \tag{10}$$

or equivalently

$$t\,\mathrm{d}E = A(t)\Gamma(t) \tag{11}$$

which is shown to be integrability of the following linearisation equations:

$$\mathrm{d}F(t) = \Gamma(t)F(t) \tag{12}$$

where $F(t)$ is a 2×2 matrix function of t, ξ and η, where t is a parameter and

$$A(t) = I - t(E + E^+) \tag{13}$$

$$\Gamma(t) = t[1 - 2t(\beta + \alpha^*)]^{-1}\,\mathrm{d}E. \tag{14}$$

Without loss of generality, we select some auxiliary conditions on $F(t)$

$$F(0) = I \tag{15}$$

$$\dot{F}(0) = E \tag{16}$$

$$F(t)^\times A(t)F(t) = I \tag{17}$$

$$\det F(t) = \lambda^{-1}(t) = [(1 - 2\beta t)^2 - (2\alpha t)^2]^{-1/2} \tag{18}$$

where $\dot{F}(t) = (\partial/\partial t)F(t)$, $F(t)^\times = F^+(\bar{t})$ and $\bar{t}$ is the complex conjugate of t.

On the other hand, the usual Lax pair has the following form:

$$\begin{aligned} \partial_\xi U(\gamma) &= -\tfrac{1}{2}(1 - 1/\gamma)(1/\alpha^2)N\,\partial_\xi N U(\gamma) \\ \partial_\eta U(\gamma) &= -\tfrac{1}{2}(1 - \gamma)(1/\alpha^2)N\,\partial_\eta N U(\gamma) \end{aligned} \tag{19}$$

where $U(\gamma)$ is a Hermitian matrix and γ a parameter which is connected to t by

$$\gamma = -\frac{1 - \alpha f(t)}{1 + \alpha f(t)} = \left(\frac{1 - 2t(\beta + \alpha)}{1 + 2t(\beta - \alpha)}\right)^{1/2}. \tag{20}$$

Comparing (12) with (19), we find that both are equivalent if

$$F(t) = \left(\frac{\lambda(t)}{1 - \alpha^2 f^2(t)}\right)^{1/2}(f(t)N + I)U(\gamma). \tag{21}$$

In the following discussion we shall demonstrate that the new linearisation equations have more advantages than the old since the Kac–Moody symmetry and the Virasoro symmetry are explicitly contained in the new but not in the old. This is why we introduce the new linearisation equations instead of using the usual ones. In this way, we can further understand the similarity of the O(3) σ-model and the axially symmetric stationary Einstein field equations [12].

Let us now take account of the infinite-dimensional symmetries hidden in the model. As for the stationary axially symmetric Einstein field equations [6, 7], we give two infinitesimal transformations

$$L(s)E=-\dot{F}(s)F^{-1}(s) \tag{22}$$

and

$$T^a(s)E=-(\mathrm{i}/s)(F(s)\sigma_a F^{-1}(s)-\sigma_a) \tag{23}$$

where $F(s)$ satisfies (12). For convenience of discussion, the infinitesimal constants are deleted here. We prove that the transformations (22) and (23) are both the symmetric transformations of (10). For simplicity we give only a sketch of the proof that (10) remains invariant under the transformation (22), i.e.

$$\begin{aligned}2(\beta+\alpha^*)\,\mathrm{d}(L(s)E)&+2(L(s)\beta+L(s)\alpha^*)\,\mathrm{d}E\\&=(E+E^+)\,\mathrm{d}(L(s)E)+(L(s)E+L(s)E^+)\,\mathrm{d}E.\end{aligned} \tag{24}$$

Firstly it is necessary to transform α and β. From $\beta=\frac{1}{2}\,\mathrm{Re}(\mathrm{Tr}\,E)$ we have

$$\begin{aligned}L(s)\beta&=-\tfrac{1}{2}\,\mathrm{Re}(\mathrm{Tr}\,\dot{F}(s)F^{-1}(s))=\tfrac{1}{2}\lambda^{-1}(s)\dot{\lambda}(s)\\&=-\frac{\beta(1-2s\beta)+2s\alpha^2}{(1-2s\beta)^2-(2s\alpha)^2}.\end{aligned} \tag{25}$$

By using (2), (13) and (17), we obtain

$$L(s)\alpha=-\frac{\alpha}{(1-2s\beta)^2-(2s\alpha)^2}. \tag{26}$$

Evaluating the exterior derivative of (22) and exploiting

$$2(\beta+\alpha^*)\Gamma(s)=(E+E^+)\Gamma(s)=-\dot{A}(s)\Gamma(s)$$

we see that

$$2(\beta+\alpha^*)\,\mathrm{d}(L(s)E)=(E+E^+)\,\mathrm{d}(L(s)E)-[E+E^+,\dot{F}(s)F^{-1}(s)]\Gamma(s) \tag{27}$$

and

$$2(L(s)\beta+L(s)\alpha^*)\,\mathrm{d}E=(1/s)\dot{A}(s)\Gamma(s). \tag{28}$$

In terms of (13) and (17), it is easy to derive

$$\begin{aligned}(L(s)E+L(s)E^+)\,\mathrm{d}E&=-(\dot{F}(s)F^{-1}(s)+F^{-1}(s)^\times\dot{F}(s)^\times)(1/s)A(s)\Gamma(s)\\&=-[E+E^+,\dot{F}(s)F^{-1}(s)]\Gamma(s)+(1/s)\dot{A}(s)\Gamma(s).\end{aligned} \tag{29}$$

We demonstrate the identity of (24) after summarising (27)-(29).

It is apparent that α and β play an important role in exploring the symmetry discussed above because if α and β do not occur the symmetry will be broken down. This is the reason that we previously introduced α and β. However, α and β become trivial under the transformation (23) since

$$T^a(s)\alpha=T^a(s)\beta=0.$$

We should consider the variance of $N(x)$ under both the transformations (22) and (23). By means of the relations (20) and (21), we have

$$L(s)N=-\lambda^2(s)N-\frac{f(s)}{1-\alpha^2f^2(s)}\frac{\mathrm{d}\gamma}{\mathrm{d}s}\left[N,\frac{\mathrm{d}}{\mathrm{d}\gamma}U(\gamma)U^{-1}(\gamma)\right] \tag{30}$$

and

$$T^a(s)N=\frac{f(s)}{s(1-\alpha^2f^2(s))}[N, U(\gamma)\mathrm{i}\sigma_a U^{-1}(\gamma)] \tag{31}$$

which can be used to generate the infinite conservation currents from the Lagrangian of the system. Similar transformations appear in [9] except for lacking the first term of (30) and the coefficients of the rest terms. However, the previous transformations, unlike our transformations, do not possess the properties of the infinite-dimensional Lie algebras.

Furthermore, we substitute the transformations (22) and (23) into (12) to obtain the corresponding transformations of $F(t)$,

$$L(s)F(t)=-\frac{t}{t-s}(t\dot{F}(t)F^{-1}(t)-s\dot{F}(s)F^{-1}(s))F(t) \tag{32}$$

and

$$T^a(s)F(t)=-\frac{\mathrm{i}t}{t-s}(F(t)\sigma_aF^{-1}(t)-F(s)\sigma_aF^{-1}(s))F(t) \tag{33}$$

respectively, which are compatible with auxiliary conditions (15)-(18).

Expanding $L(s)$ and $T^a(s)$ in powers of

$$L(s)=\sum_{k=0}^{\infty} L_ks^k \qquad T^a(s)=\sum_{k=0}^{\infty} T^a_ks^k \tag{34}$$

we express the transformations (32) and (33) in the infinitesimal Riemann-Hilbert transformations

$$L_kF(t)F^{-1}(t)=-\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\frac{ty^{-k+1}\dot{F}(y)F^{-1}(y)}{y(y-t)}\,\mathrm{d}y \qquad k\geqslant 0 \tag{35}$$

and

$$F^a_kF(t)F^{-1}(t)=-\frac{1}{2\pi}\int_{C_{0,t}}\frac{ty^{-k}F(y)\sigma_aF^{-1}(y)}{y(y-t)}\,\mathrm{d}y \qquad k\geqslant 0 \tag{36}$$

where $C_{0,t}$ represents a circle C surrounding poles at $y=0, t$.

By using (35) and (36), we can now calculate the following commutators:

$$[L_m, L_n]F(t)=(m-n)L_{m+n}F(t) \tag{37}$$

$$[L_m, T^b_n]F(t)=-nT^b_{m+n}F(t) \tag{38}$$

$$[T^a_m, T^b_n]F(t)=2\mathrm{i}\varepsilon_{abc}T^c_{m+n}F(t). \tag{39}$$

Obviously, the infinite series of operators L_m and T^a_n form the semidirect product of the Virasoro algebra and the Kac-Moody algebra, and the generating function $F(t)$ provides the representations of the semidirect product algebra which are non-unitary and non-highest weight as mentioned before.

Since a similar procedure to the proof of the commutator (39) can be found in [13] for the two-dimensional Heisenberg model we need only verify the commutator (37). Starting with

$$L_m\dot{F}(t)=-\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\frac{y^{-m+1}}{(y-t)^2}\dot{F}(y)F^{-1}(y)\,\mathrm{d}y\,F(t)-\frac{t}{2\pi\mathrm{i}}\int_{C_{0,t}}\frac{y^{-m+1}}{y-t}\dot{F}(y)F^{-1}(y)\,\mathrm{d}y\,\dot{F}(t)$$

we have

$$
\begin{aligned}
&([L_m, L_n]F(t))F^{-1}(t)\\
&\quad=\frac{t}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{1}{y(y-t)}\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t,y}}\mathrm{d}z\bigg(\frac{y^{-m+2}z^{-n+1}}{z(z-y)}\\
&\qquad\times[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]-\frac{y^{-n+2}z^{-m+1}}{z(z-y)}[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]\\
&\qquad+\frac{ty^{-m+1}z^{-n+1}}{z(z-t)}[\dot F(y)F^{-1}(y),\dot F(z)F^{-1}(z)]\bigg)\\
&\qquad+\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{t}{y(y-t)}\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t,y}}\mathrm{d}z\frac{1}{(z-y)^2}(y^{-m+1}z^{-n+1}\\
&\qquad-y^{-n+1}z^{-m+1})\dot F(z)F^{-1}(z)\\
&\quad=\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t,y}}\mathrm{d}z\bigg(\frac{ty^{-m+1}z^{-n+1}}{y(z-y)(z-t)}[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]\\
&\qquad-\frac{ty^{-n+1}z^{-m+1}}{z(z-y)(y-t)}[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]\bigg)\\
&\qquad+\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{t}{y(y-t)}\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t,y}}\mathrm{d}z\frac{1}{(z-y)^2}(y^{-m+1}z^{-n+1}\\
&\qquad-y^{-n+1}z^{-m+1})\dot F(z)F^{-1}(z)\\
&\quad=\frac{1}{2\pi\mathrm{i}}\int_{C_0'}\mathrm{d}y\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}z\frac{ty^{-m+1}z^{-n+1}}{y(z-y)(z-t)}[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]\\
&\qquad-\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{1}{2\pi\mathrm{i}}\int_{C_0'}\mathrm{d}z\frac{ty^{-n+1}z^{-m+1}}{z(z-y)(y-t)}[\dot F(z)F^{-1}(z),\dot F(y)F^{-1}(y)]\\
&\qquad+\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t}}\mathrm{d}y\frac{1}{2\pi\mathrm{i}}\frac{t}{y(y-t)}\int_{C_{0,t,y}}\mathrm{d}z\frac{1}{(z-y)^2}(y^{-m+1}z^{-n+1}\\
&\qquad-y^{-n+1}z^{-m+1})\dot F(z)F^{-1}(z).
\end{aligned}\tag{40}
$$

For the first term at the second step, because $y=t$ is not a pole for the integrand, the circle $C_{0,t}$ is selected as a circle C_0' lying inside $C_{0,t}$, and $C_{0,t,y}$ as $C_{0,t}$. In the second term, $C_{0,t,y}$ can only be expressed as C_0' since the contribution from the pole at $z=y$ is equal to zero. At the final step the first two terms may cancel each other if y and z are interchanged in one of them. Hence, there remains the last term to be calculated. If we set

$$\dot F(z)F^{-1}(z)=\sum_{k=0}^{\infty}M^{(k)}z^k$$

then

$$\frac{1}{2\pi\mathrm{i}}\int_{C_{0,t,y}}\mathrm{d}z\frac{1}{(z-y)^2}y^{-m+1}z^{-n+1}\dot F(z)F^{-1}(z)=\sum_{l=0}^{\infty}(l-n+1)y^{l-m-n+1}M^{(l)}. \tag{41}$$

Finally we substitute the above relation into (40) to yield the commutator (37).

In our previous papers [7], we had to introduce an infinite hierarchy of the Kinnersley-Chitre potentials

$$G(t_1, t_2) = \sum_{m,n=0}^{\infty} N^{(m,n)} t_1^m t_2^n = \frac{1}{t_1 - t_2}(-t_1 + t_2 F^{-1}(t_1) F(t_2)) \tag{42}$$

to compute the commutators (37)-(39). The actions of L_k and T_k^a on the Kinnersley-Chitre potentials $N^{(m,n)}$ are, respectively,

$$L_k N^{(m,n)} = -\left((m+k) N^{(m+k,n)} + n N^{(m,n+k)} + \sum_{p=1}^{k} (k-p) N^{(m,p)} N^{(k-p,n)}\right) \qquad k \geqslant 0 \tag{43}$$

and

$$T_k^a N^{(m,n)} = -\gamma_a N^{(m+k,n)} + N^{(m,n+k)} \gamma_a + \sum_{b=1}^{k} N^{(m,p)} \gamma_a N^{(k-p,n)} \qquad k \geqslant 0,\ \gamma_a = \mathrm{i}\sigma_a. \tag{44}$$

However, our treatment in this letter avoids introducing such auxiliary quantities and our calculation becomes much simpler than before.

In the past we also indicated that the operators $L_k (k = 0, \pm 1)$ form the transformations of the Cosgrove group [14]. For the O(3) non-linear σ-model we can constitute a group in the same way. In the self-dual equations (10), there are two trivial transformations

$$(R)_\Lambda\{\alpha, \beta, E\} = \{\Lambda\alpha, \Lambda\beta, \Lambda E\} \qquad \Lambda = \mathrm{e}^{\lambda} \tag{45}$$

$$(Z)_\mu\{\alpha, \beta, E\} = \{\alpha, \beta + \mu, E + \mu I\} \tag{46}$$

and a non-trivial transformation

$$\begin{aligned}
(\tilde{Q})_t E &= F^{-1}(t) \dot{F}(t) \\
(\tilde{Q})_t \alpha &= \frac{\alpha}{(1-2t\beta)^2 - (2t\alpha)^2} \\
(\tilde{Q})_t \beta &= \frac{\beta(1-2t\beta) + 2t\alpha^2}{(1-2t\beta)^2 - (2t\alpha)^2}.
\end{aligned} \tag{47}$$

In fact, the non-trivial transformation is the dual transformation given in [9]. From (45)-(47) one can show that R, Z and $\tilde{Q}$ indeed form the group that is isomorphic to a SL(2, R) group.

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